# Note on DBI dynamics of Dbrane Near NS5-branes

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## Abstract

In this note, we investigate the homogeneous radial dynamics of (Dp, NS5)-systems without and with one compactified transverse direction, in the framework of DBI effective action. During the homogeneous evolution, the electric field on the D-brane is always conserved and the radial motion could be reduced to an one-dimension dynamical system with an effective potential. When the Dp-brane energy is not high, the brane moves in a restricted region, with the orbits depending on the conserved energy, angular momentum through the form of the effective potential. When the Dp-brane energy is high enough, it can escape to the infinity. It turns out that the conserved angular momentum plays an interesting role in the dynamics. Moreover, we discuss the gauge dynamics around the tachyon vacuum and find that the dynamics is very reminiscent of the string fluid in the rolling tachyon case.

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# 1 Introduction

The Dirac-Born-Infeld (DBI) field theory encodes the dynamics of D-brane[1]. It has been proved to be a very powerful tool to study various aspects of the D-brane dynamics. In [2], A. Sen proposed a DBI-like effective field theory to study the tachyon dynamics on the non-BPS brane. Very surprisingly, the effective field theory describe the tachyon condensation quite well[3]. This fact leads to a new kind of open-closed string duality[4].

Very recently, it has been found that the radial behavior of Dp-brane near NS5-branes is very similar to the rolling tachyon[5]. In the near throat region, the classical motion of D-brane is a hair-pin brane in CHS theory, which could be studied in the framework of BCFT. In [7], the boundary state of the supersymmetric hairpin brane has been constructed and in [7, 8] the closed string radiation of the D-brane near NS5-branes has been discussed. In [9], taking into account of the constant electromagnetic field, the radial dynamics and closed string radiation has been investigated. (see also [10]) It was found that due to the constant electric field, the closed string radiation without winding is finite. Nevertheless, the emission of the closed string with windings dominate the radiation and is divergent. These facts are reminiscent of the closed string radiation of the rolling tachyon in the presence of constant NS $B_{\mu\nu}$ field. Therefore, the (Dp, NS5)-system gives another interesting laboratory to study the string theory in a time-dependent background. Other related discussions can be found in [11].

Furthermore, in [6] D. Kutasov studied the (Dp, NS5) system with one compactified transverse direction (which we call (Dp, NS5)' system) and noticed that certain (Dp, NS5) configuration has kink solution and behaves like a non-BPS brane in type II theory. Unfortunately, the CFT analysis on this configuration seems to be out of reach. In this note, we take the philosophy that DBI effective action catch most of the tachyon dynamics and investigate the DBI dynamics of (Dp, NS5)-systems. We will concentrate on the homogeneous evolution in which case the radial motion could be directly related to the rolling tachyon. However, the introduction of the angular momentum into the system make the dynamics much richer. (see [18] for the similar discussions in Dp-anti-Dp system)

Another interesting issue is the gauge dynamics of the tachyon condensation in the DBI effective action. In the rolling tachyon case, it has been shown that two components of pressureless fluid survive the tachyon condensation: one is conserved electric flux lines, named string fluid; the other is the dust-like pressureless tachyon matter[12, 13]. Around the tachyon vacuum, the gauge dynamics could be well described by a set of fluid equations with integrability conditions[14]. Further investigation shows that such string fluid may have a closed string interpretation[16, 4, 15]. We will study this issue in the (Dp, NS5) geometric tachyon configurations.

We will revisit (Dp, NS5)-system with gauge field fluctuations and angular momentum in section 2. We study the radial dynamics of (Dp, NS5)'-system in section 3. Discussions and conclusions will be in section 4.

## 2 The Radial Dynamics of (Dp, NS5)-system

In this section we use the effective DBI action to analyze the Dp-brane dynamics near the NS5-branes. The tension of a NS5-brane $\sim 1/g_s^2$ while the tension of a $Dp$-brane $\sim 1/g_s$, so it is natural to take the NS5-branes' supergravity solution as the background when the string coupling is weak.

The coordinates on the world-volume of $k$ coincident NS5-branes are $x^\mu$, $\mu = 0, 1, ...5$, and we use $x^n, n = 6, 7, 8, 9$ to label the four transverse dimensions. Let the Dp-brane be parallel to NS5-branes and let the world-volume of Dp-brane lie along $x^0, \cdots x^p$ with $2 \leq p < 5$. Such a system breaks supersymmetry completely and is unstable.

Setting $r^2 = \sum_{n=6}^{9} x^n x^n$, the low energy supergravity solution of NS5-branes is

$$\begin{aligned} ds^2 &= dx^\mu dx_\mu + H(r) dx^n dx_n \equiv g_{MN} dx^M dx^N \\ \frac{g_s(\Phi)}{g_s} &= \exp(\Phi - \Phi_0) = \sqrt{H(r)} \\ H_{mnp} &= -\epsilon_{qmnp} \partial^q \Phi, \\ H(r) &= 1 + \frac{k l_s^2}{r^2}, \end{aligned} \tag{1}$$

where $H_{mnp}$ is the NS 2-form field strength, $g_s$ is the asymptotic string coupling, and $l_s$ is the string length unit.

### 2.1 Dynamics without angular momentum

Since there is an $SO(4)$ rotational symmetry for the four transverse dimensions, the angular momentum is conserved. Let's temporally ignore the angular momentum and focus on the radial motion of the bounded system first. The dynamics of Dp-brane in the NS5-branes background is well described by the DBI action

$$S_p = -\tau_p \int d^{p+1} x e^{-(\Phi - \Phi_0)} \sqrt{-det(G_{\mu\nu} + B_{\mu\nu} + F_{\mu\nu})} \tag{2}$$

where $\tau_p$ is the asymptotic tension of the Dp-brane

$$\tau_p \sim \frac{1}{g_s l_s^{p+1}} \tag{3}$$

There is a U(1) gauge freedom which can be used to set $A_0 = 0$. We are free to set $B_{\mu\nu} = 0$ due to the gauge transformation and we have:

$$-Det(G + F) = Det(h)(1 - H\dot{r}^2) - E_i^+ D_{ik} E_k^- \tag{4}$$

$$E_i^\pm = F_{0i} \pm H\dot{r}\partial_i r = E_i \pm H\dot{r}\partial_i r \tag{5}$$

$$h_{ij} = \delta_{ij} + F_{ij} + H\partial_i r \partial_j r \tag{6}$$

$$D_{ij} = (-1)^{i+j} \Delta_{ji}(h) = Det(h) h_{ij}^{-1} \tag{7}$$

Then the action is written as

$$-\tau_p \int dt \frac{1}{\sqrt{H}} \sqrt{Det(h)(1 - H\dot{r}^2) - E_i^+ D_{ik} E_k^-} \tag{8}$$

From this we can evaluate the canonical momentum conjugate to $A_i$ and $r$:

$$\mathbf{\Pi}_r = \frac{\tau_p}{\sqrt{H}\sqrt{Det(h)(1-H\dot{r}^2) - E_i^+ D_{ik} E_k^-}}(Det(h)H\dot{r} - \frac{E_k^+ D_{ki} - D_{ik}E_k^-}{2}H\partial_i r) \tag{9}$$

$$\mathbf{\Pi}_e^i = \frac{\tau_p}{\sqrt{H}\sqrt{Det(h)(1-H\dot{r}^2) - E_i^+ D_{ik} E_k^-}}(\frac{E_k^+ D_{ki} + D_{ik}E_k^-}{2}) \tag{10}$$

And the conserved Hamiltonian is

$$\begin{aligned}
\mathcal{H} &= \frac{\tau_p Det(h)}{\sqrt{H}\sqrt{Det(h)(1-H\dot{r}^2) - E_i^+ D_{ik} E_k^-}} \\
&= \sqrt{|\vec{\mathbf{\Pi}}_e|^2 + \frac{1}{H}\mathbf{\Pi}_r^2 + H(\mathbf{\Pi}_e^i \partial_i r)^2 + |\vec{\mathcal{P}}|^2 + \frac{Det(h)}{H}} \qquad (11)\\
\mathcal{P}_i &= -F_{ik}\mathbf{\Pi}_e^k - \partial_i r \mathbf{\Pi}_r \qquad (12)
\end{aligned}$$

In the above $\mathcal{P}$ is the conserved Noether charge associated with the spatial translation along the world volume.

As remarked in [5], after the field redefinition, one may treat $r$ as the rolling tachyon field and $1/\sqrt{H}$ as the tachyon potential. As the geometric tachyon field $r \to 0$, the tachyon potential tends to zero, namely as D-brane falls down close to the NS5-branes, $H \to \infty$. From (11), the last term in it vanishes, and the term $H(\mathbf{\Pi}_e^i \partial_i r)^2$ dominates. So the energy conservation requires that $\partial_i r \approx 0$ so the Dp-brane inclines to homogenously evolve in the near throat region.

Now we treat the motion of the Dp-brane as homogeneous radial evolution, that is all the world volume fields are only functions of $X^0$ or $t$, so the only non-vanishing components of $F_{\mu\nu}$ is $F_{0i} = -F_{i0} = \dot{A}_i = e_i$. The pull-back metric and the world-volume gauge field is

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} -1+H\dot{r}^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{F}_{\mu\nu} = \begin{pmatrix} 0 & e_1 & e_2 & e_3 \\ -e_1 & 0 & 0 & 0 \\ -e_2 & 0 & 0 & 0 \\ -e_3 & 0 & 0 & 0 \end{pmatrix}. \tag{13}$$

And the DBI action reads

$$S_p = -\tau_p \int d^{p+1}x \frac{1}{\sqrt{H}}\sqrt{1-H\dot{r}^2 - e^2} = -\tau_p V \int dt \frac{1}{\sqrt{H}} K$$
$$K = \sqrt{1-H\dot{r}^2 - e^2} \ , e^2 = \sum_{i=1}^{p} e_i^2 \tag{14}$$

where $V$ is the volume of the D-brane.

The canonical momentum density conjugate to $r$ and $A_i$ are as follows:

$$\mathbf{\Pi}_r = \tau_p \frac{\sqrt{H}}{K}\dot{r}$$
$$\mathbf{\Pi}_e^i = \tau_p \frac{e_i}{\sqrt{H}K} = n_i \tag{15}$$

and the conserved Hamiltonian is

$$\mathcal{H}=\mathbf{\Pi}_r\dot{r}+\mathbf{\Pi}_e^i e_i-\mathcal{L}_{DBI}=\frac{\tau_p}{\sqrt{H}K}=E \tag{16}$$

We can obtain the strength tensor $T_{\mu\nu}$ and NS source tensor $S_{\mu\nu}$ as in [9]

$$\begin{aligned}\delta S &= -\frac{\tau_p}{2}e^{-(\Phi-\Phi_0)}\sqrt{-det(\mathbf{G}+\mathbf{B})}(\mathbf{G}+\mathbf{B})^{\mu\nu}(\delta g_{\mu\nu}+\delta b_{\mu\nu}) \\ &= -\frac{\tau_p}{2}\frac{1}{\sqrt{H(r)}}\sqrt{-det(\mathbf{G}+\mathbf{B})}(\mathbf{G}+\mathbf{B})^{\mu\nu}(\delta g_{\mu\nu}+\delta b_{\mu\nu})\end{aligned}$$

The result is:

$$\mathbf{T}_{\mu\nu}=\frac{\tau_p}{\sqrt{H}K}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & e_1^2-K^2 & -e_1e_2 & -e_1e_3\\ 0 & -e_1e_2 & e_2^2-K^2 & -e_2e_3\\ 0 & -e_1e_3 & -e_2e_3 & e_3^2-K^2\end{pmatrix},\mathbf{S}_{\mu\nu}=\frac{\tau_p}{\sqrt{H}K}\begin{pmatrix}0 & -e_1 & -e_2 & -e_3\\ e_1 & 0 & 0 & 0\\ e_2 & 0 & 0 & 0\\ e_3 & 0 & 0 & 0\end{pmatrix}. \tag{17}$$

The equation of motion can be written as an one-dimensional problem effectively:

$$\dot{r}^2=-V_{eff}=\frac{1}{H}-\frac{\tau_p^2+Hn^2}{H^2E^2} \tag{18}$$

We can classify the motion of the D-brane according to the behavior of $V_{eff}$. Set $a=\frac{\tau_p^2}{E^2}$, $b=\frac{n^2}{E^2}\leq 1$, then

$$V_{eff}=\frac{a}{H^2}+\frac{b-1}{H}. \tag{19}$$

We have

i. when $\frac{1-b}{a}<1$, or $E^2<n^2+\tau_p^2$, the D-brane move between $r=0$ and $r=r_{max}$ where

$$r_{max}=\sqrt{k}\sqrt{\frac{E^2-n^2}{n^2+\tau_p^2-E^2}}l_s \tag{20}$$

ii. when $\frac{1-b}{a}\geq 1$, or $E^2\geq n^2+\tau_p^2$, the brane could escape to infinity.

In the near throat region, one has $H=\frac{kl_s^2}{r^2}$ and one can solve the equations of motions and get a solution as the modified hair-pin brane.

In the above we set the conserved quantity $\mathbf{\Pi}_e^i$ to $n_i$, which is the electric flux long the direction $x^i$ on the D-brane. In the usual discussion of the DBI action with gauge field, the conserved quantity is only the electric flux $n_i$ rather than the electric field $e_i$[12]. However as a direct consequence of homogeneity, the electric field $e_i$ is also conserved in our case:

$$e_i=\frac{n_i}{E} \tag{21}$$

so that the problem is reduced to the case studied in some details in [9]. The energy conservation requires that when the D-brane approaches very closely to the NS5-branes, the geometric tachyon reaches its vacuum

and $H \to \infty$, so one has $K = 0$, i.e. $H\dot{r}^2 = 1 - e^2$. When the electric field takes the critical value $e = 1$, the open-string degrees of freedom on the D-brane decouples from the bulk closed string modes, the geometric tachyon does not roll and there is no closed string radiation from the D-brane and no tachyon matter. When $e < 1$, the behavior of the D-brane in the near throat region of NS5-background turns out to be a modified hairpin solution. The presence of the electric field slows down the rolling of the geometric tachyon and in the end of the tachyon condensation, besides the usual tachyon matter, there exist the fluid of electric flux lines. This could be seen from the partition of the energy at the vacuum:

$$\mathcal{H} = E = \sqrt{\mathbf{\Pi}_e^2 + \frac{\mathbf{\Pi}_r^2}{H}} \tag{22}$$

where

$$\mathbf{\Pi}_e^2 = \sum_i n_i^2 = e^2 E^2, \qquad \frac{\mathbf{\Pi}_r^2}{H} = (1 - e^2)E^2 \tag{23}$$

are the flux energy and kinetic energy respectively. The partition of the energy is reminiscent of the vacua $\dot{T}^2 + e^2 = 1$ in the rolling tachyon case[13]. Therefore, when Dp-brane fall towards the NS5-branes, it loses energy by closed string radiation but the constant electric field slows its motion and survive the tachyon condensation as the string fluid.

One may investigate the fluctuations around the tachyon vacuum from the Hamiltonian dynamics[13]. Under the identification

$$\partial_i T = \sqrt{H}\partial_i r, \qquad \mathbf{\Pi}^T = \frac{\mathbf{\Pi}_r}{\sqrt{H}}, \tag{24}$$

and take $T$ as a component of gauge field along an imagined direction

$$A_T = T, \tag{25}$$

we may rewrite the Hamiltonian (11) at the end of geometric tachyon condensation as

$$\mathcal{H} = \sqrt{\mathbf{\Pi}^I \mathbf{\Pi}^I + (F_{IJ}\mathbf{\Pi}^J)^2}, \tag{26}$$

where $\partial_T \equiv 0$ on any object and $I, J = 1, 2, \cdots p$ and $T$. This relation is exactly the same as the one in the rolling tachyon case. Correspondingly, one has the same Hamiltonian equations of motions. And the discussion of the fluctuations around the constant background $\mathbf{\Pi}_0^M$ follows the same line. It turns out the dynamical fluctuations obey the equation

$$(\partial_t^2 - \sum_{i=1}^{q} e_i^2 \partial_i^2)\delta\mathbf{\Pi}^M = 0, \tag{27}$$

where we assume that the electric fields appear in $q$ directions. After proper rescaling $x_i' = \frac{x_i}{e_i}$, the above equation could be transformed to the wave function in $q$ spatial dimensions. The general solution to the equation is

$$f_L(t - \vec{k} \cdot \vec{r}', x_j) + f_R(t + \vec{k} \cdot \vec{r}', x_j) \tag{28}$$

where $\vec{k}$ is any unit vector and $x_j$ is the directions without electric field. When all $e_i$ vanish, the solution shows the Carrollian behavior discussed in [13]. When $q = 1$, the solution reduces to the one where $p$ degrees of freedom propagate along $\vec{e}$ at speed $e$.

From the above discussion, it is obvious that after field definition, the geometric tachyon condensation problem in (Dp, NS5)-system could be identified with the rolling tachyon problem on a non-BPS brane. Such identification works not only in the pure tachyon case, it also works in the case when the tachyon coupled to the worldvolume gauge fields. In [14], the classical dynamics of the tachyon field coupled to the gauge field has been studied carefully, in the framework of Hamiltonian dynamics. It could be shown that the canonical equations of motions and the conservation of the energy-momentum tensor give us the fluid equations of motions

$$\begin{aligned} \partial_0 n^I + v^i \partial_i n^I &= n^i \partial_i v^I \\ \partial_0 v^I + v^i \partial_i v^I &= n^i \partial_i n^I \end{aligned} \tag{29}$$

with the integrability condition

$$v^i = -F_{ij} n^j - n^T \partial_i T, \qquad v^T = \partial_i T n^i, \tag{30}$$

where

$$n^i = \frac{\mathbf{\Pi}^i}{\mathcal{H}}, \qquad n^T = \frac{\mathbf{\Pi}^T}{\mathcal{H}} \tag{31}$$

$$v^i = \frac{F_{ij}\mathbf{\Pi}^j}{\mathcal{H}}, \qquad v^T = \frac{\partial_i T \mathbf{\Pi}^i}{\mathcal{H}} \tag{32}$$

satisfying

$$n^I n^I + v^I v^I = 1, \qquad n^I v^I = 0. \tag{33}$$

The various classical solutions representing the distribution of the tachyon matter and string fluid can be obtained as in [14]. Even though there is tight coupling between string fluid and tachyon matter fluid, the distribution of the two fluid components in the static configuration, which has $\partial_0 = 0, F_{ij}\mathbf{\Pi}^j = 0$, is almost independent with each other and the only requirement is that the static distribution of the tachyon matter must be stay along the string fluid direction. For example, the case we discussed above corresponds to the electric flux densities along $n^i$ which have electric fields, and nonvanishing tachyon matter $\mathbf{\Pi}$. Moreover, if there exist a momentum density, the evolution of the geometric tachyon is not homogeneous. For instance, one may allow a boost along $x^1$ to induce a momentum $v^1 = -n^T \partial_1 T$ without turning on magnetic field. This implies that $\partial_1 T \neq 0$. From the field redefinition, approximately one has

$$r = \exp\left(-\frac{v^1}{\sqrt{(1-e^2)k}l_s} x^1\right). \tag{34}$$

In other words, the inhomogeneous evolution of the radial direction is suppressed in the near horizon region.

## 2.2 Dynamics with angular momentum

Now we consider the motion of the bounded system with angular momentum. By using the transverse $SO(4)$ symmetry, the orbit can be confined on the $X^6, X^7$ plane. Set

$$X^6 = R\cos\theta\ , \qquad X^7 = R\sin\theta, \tag{35}$$

the DBI action then reads

$$S_p = -\tau_p \int d^{p+1}x \frac{1}{\sqrt{H}} \sqrt{1 - H(\dot{R}^2 + R^2\dot{\theta}^2) - e^2} = -\tau_p V \int dt \frac{K}{\sqrt{H}} \tag{36}$$

where $K = \sqrt{1 - H(\dot{R}^2 + R^2\dot{\theta}^2) - e^2}$.

Now the canonical momentum conjugate to $R$, $\theta$, $A_i$ are

$$\begin{aligned} \mathbf{\Pi}_R &= \frac{\delta S_p}{\delta \dot{R}} = \tau_p \frac{\sqrt{H}\dot{R}}{K} \\ \mathbf{\Pi}_\theta &= \frac{\delta S_p}{\delta \dot{\theta}} = \tau_p \frac{\sqrt{H}}{K} R^2 \dot{\theta} = L \\ \mathbf{\Pi}_e^i &= \frac{\delta S_p}{\delta \dot{A}_i} = \tau_P \frac{e_i}{\sqrt{H} K} = n_i \end{aligned} \tag{37}$$

and the Hamiltonian is

$$\mathcal{H} = \mathbf{\Pi}_R \dot{R} + \mathbf{\Pi}_\theta \dot{\theta} + \mathbf{\Pi}_e^i e_i - \mathcal{L}_{DBI} = E = \frac{\tau_p}{\sqrt{H} K} \tag{38}$$

In the above we have set the conserved quantity $\mathbf{\Pi}_\theta$, $\mathbf{\Pi}_e^i$, $\mathcal{H}$ as $L$, $n_i$, and $E$ respectively.

Similarly, the equation of motion of $R$ is reduced to an one-dimensional problem:

$$\dot{R}^2 = -V_{eff} = \frac{1}{H} - \frac{1}{H^2 E^2}(\tau_p^2 + \frac{L^2}{R^2} + n^2 H) \tag{39}$$

where $n^2 = \sum_{i=1}^{p} n_i^2$.

When $R$ is large compared to $l_s$, one could approximate $H^{-1}$ and get

$$-V_{eff} = (1 - e^2 - \frac{\tau_p^2}{E^2}) + \frac{1}{R^2}((\frac{2\tau_p^2}{E^2} - 1 + e^2) k l_s^2 - \frac{L^2}{E^2}). \tag{40}$$

Therefore, it is easy to figure out that

1. if $E^2 < n^2 + \tau_p^2$, and
$$\frac{L^2}{E^2} < (\frac{2\tau_p^2}{E^2} - 1 + e^2) k l_s^2, \tag{41}$$
then $R \in [0, 1/u_0]$, where $u_0$ is the positive root of $V_{eff} = 0$; if $L$ is larger, the radial motion is frozen.

2. if $n^2 + 2\tau_p^2 > E^2 \geq n^2 + \tau_p^2$, D-brane move between $R = 0$ and infinity;

3. if $E^2 > n^2 + 2\tau_p^2$ or $n^2 + 2\tau_p^2 > E^2 \geq n^2 + \tau_p^2$ but $L$ is large, there is a potential barrier for the D-brane to reach $R = 0$, $R \in [1/u_0, \infty]$, where $u_0$ is the positive root of $V_{eff} = 0$.

Obviously, in order to have bounded solution, we need to require $E^2 < n^2 + \tau_p^2$.

When the D-brane reaches the near horizon region, one has

$$\dot{R}^2 = (\frac{1 - e^2}{k l_s^2} - \frac{L^2}{k^2 l_s^4 E^2}) R^2 - \frac{\tau_p^2}{k^2 l_s^4 E^2} R^4 \tag{42}$$

and the solution is

$$r = \frac{\beta}{\cosh \alpha t} \tag{43}$$

where

$$\alpha^2 = \frac{1}{kl_s^2}(1 - e^2 - \frac{L^2}{kl_s^2 E^2}) \ , \quad \beta^2 = \frac{kl_s^2 E^2}{\tau_p^2}(1 - e^2 - \frac{L^2}{kl_s^2 E^2}). \tag{44}$$

In order to have bounded solution, beside requiring $E^2 < n^2 + \tau_p^2$, one has also require

$$\frac{L}{E} < \sqrt{(1 - e^2)} k l_s. \tag{45}$$

The existence of the angular momentum will slow down the falling of the Dp-brane to the NS5-branes, just like the role played by the constant electric field.

To study the remnants after tachyon condensation, we rewrite the Hamiltonian as

$$\mathcal{H} = \sqrt{\mathbf{\Pi}_e^2 + \frac{\mathbf{\Pi}_R^2}{H} + \frac{\mathbf{\Pi}_\theta^2}{R^2 H} + \frac{\tau_p^2}{H}}. \tag{46}$$

At the end of tachyon condensation, $H \to \infty$, the potential vanishes. The novel feature here is that the angular momentum do contribute to the energy since $R^2 H = kl_s^2$. Besides the string fluid and tachyon matter, the angular momentum survives the tachyon condensation. The kinetic energy of geometric tachyon is

$$\frac{\pi_R^2}{H} = (1 - e^2)E^2 - \frac{L^2}{kl_s^2}, \tag{47}$$

which implies (45). In this case, due to the existence of the angular momentum, the electric field take its critical value at

$$e_c = 1 - \frac{L^2}{kl_s^2 E^2} \tag{48}$$

when the radial motion is frozen.

It is remarkable that even with the existence of the angular momentum, the remnants of the tachyon condensation is still pressureless fluid. If the D-brane initially has nonzero angular momentum, it will spirally fall to the NS5-branes. During the falling, it radiate closed string modes and lose energy. The electric field is still constant during homogeneous evolution and somehow slows down the evolution.

In order to see the influence of the angular momentum to the gauge dynamics, let us analyze the canonical formulation of the DBI action more carefully. Now the action is

$$S_p = -\tau_p \int dt \frac{1}{\sqrt{H}} \sqrt{Det(h)(1 - H(\dot{R}^2 + R^2\dot{\theta}^2) - E_i^+ D_{ik} E_k^-}, \tag{49}$$

with

$$E_i^{\pm} = E_i \pm H(\dot{R}\partial_i R + R^2 \dot{\theta} \partial_i \theta) \tag{50}$$

$$h_{ij} = \delta_{ij} + F_{ij} + H(\partial_i R \partial_j R + R^2 \partial_i \theta \partial_j \theta) \tag{51}$$

$$D_{ij} = Det(h) h_{ij}^{-1}. \tag{52}$$

The canonical momenta conjugate to $R$ and $\theta$ are

$$\mathbf{\Pi}_R = \frac{\tau_p}{\sqrt{H}\sqrt{Det(h)(1 - H(\dot{R}^2 + R^2\dot{\theta}^2) - E_i^+ D_{ik} E_k^-}}(Det(h)H\dot{R} - \frac{E_k^+ D_{ki} - D_{ik} E_k^-}{2} H\partial_i R) \quad (53)$$

$$\mathbf{\Pi}_\theta = \frac{\tau_p}{\sqrt{H}\sqrt{Det(h)(1 - H(\dot{R}^2 + R^2\dot{\theta}^2) - E_i^+ D_{ik} E_k^-}}(Det(h)HR^2\dot{\theta} - \frac{E_k^+ D_{ki} - D_{ik} E_k^-}{2} HR^2\partial_i\theta) \quad (54)$$

And the conserved energy density and momentum density are

$$\mathcal{H} = \sqrt{|\vec{\mathbf{\Pi}}_e|^2 + \frac{\mathbf{\Pi}_R^2}{H} + \frac{\mathbf{\Pi}_\theta^2}{HR^2} + H(\mathbf{\Pi}_e^i \partial_i R)^2 + HR^2(\mathbf{\Pi}_e^i \partial_i \theta)^2 + |\vec{\mathcal{P}}|^2 + \frac{Det(h)}{H}} \quad (55)$$

$$\mathcal{P}_i = -F_{ik}\mathbf{\Pi}_e^k - \partial_i R \mathbf{\Pi}_R - \partial_i \theta \mathbf{\Pi}_\theta. \quad (56)$$

Let us introduce a new set of fields

$$\partial_i T = \sqrt{H}\partial_i R \qquad \mathbf{\Pi}^T = \frac{\mathbf{\Pi}_R}{\sqrt{H}} \quad (57)$$

$$\partial_i S = \sqrt{H} R \partial_i \theta \qquad \mathbf{\Pi}^S = \frac{\mathbf{\Pi}_\theta}{\sqrt{H}R}. \quad (58)$$

The field $T$ could be identified with the tachyon field and the field $S$ differs from $\theta$ by a constant rescaling in the near horizon region. Similar to the case without angular momentum, one could take $T$ and $S$ as the new components of gauge field along two imagined directions:

$$A_T = T, \qquad A_S = S. \quad (59)$$

The Hamiltonian near the tachyon vacuum takes the same form as (26) but now $I, J = 1, 2, \cdots, p$ and $T, S$. Therefore, the study of the fluctuations around the tachyon vacuum is very similar to the one in the case without angular momentum. The only difference is that apart from the fluctuations of the geometric tachyon direction $R$, one has also the angular fluctuations. Similarly, one may treat the Hamiltonian dynamics of the system with angular momentum around the vacuum as the fluid equations. The fluid equations look like (29), and the integrability condition is now

$$v^i = -F_{ij}n^j - n^T\partial_i T - n^S \partial_i S, \quad v^T = \partial_i T n^i, \quad v^S = \partial_i S n^i \quad (60)$$

where

$$n^S = \frac{\mathbf{\Pi}^S}{\mathcal{H}}, \qquad v^S = \frac{\partial_i S \mathbf{\Pi}^i}{\mathcal{H}}. \quad (61)$$

The static solutions describe the various distributions of the tachyon matter, string fluid and angular momentum. The explicit solutions can be obtained straightforwardly. If one allows momentum density along a direction, say $x^1$, the evolution of the tachyon could not be completely homogeneous. Let us turn off the magnetic field and boost along $x^1$. In this case, one has

$$v^1 = -\frac{\mathbf{\Pi}_R}{\mathcal{H}}\partial_i R - \frac{\mathbf{\Pi}_\theta}{\mathcal{H}}\partial_i\theta. \quad (62)$$

Near the vacuum, $\mathbf{\Pi}_R \propto \sqrt{H}$ so the inhomogeneous evolution of the radial direction is greatly suppressed, as in the case without angular momentum. On the other hand, $\partial_i\theta$ could be finite.

Effectively, in this geometric tachyon setup, one may combine $R$ and $\theta$ into a complex tachyon field. However due to the geometric nature, there exist a conserved angular momentum, which changes the dynamics a little bit.

## 3 The Radial Dynamics of (Dp, NS5)' System

Now let us turn to another interesting case when one of the transverse direction of NS5-Dp system is compactified on a circle of radius $r_0$. We parameterize the compactified dimension by $y$, and the three uncompactified dimensions by $r$, $\theta$, $\phi$.

Now the metric is

$$\begin{aligned} ds^2 &= dx^\mu dx_\mu + H(r) dx^n dx_n \equiv g_{MN} dx^M dx^N \\ \frac{g_s(\Phi)}{g_s} &= \exp(\Phi - \Phi_0) = \sqrt{H(r)} \\ H(r) &= 1 + \frac{k l_s^2 \sinh \frac{r}{r_0}}{2 r r_0 (\cosh \frac{r}{r_0} - \cos \frac{y}{r_0})}, \end{aligned} \tag{63}$$

We are interested in the homogeneous evolution, so the action is

$$\begin{aligned} S &= -\tau_p V \int dt \frac{1}{\sqrt{H(y,r)}} \sqrt{1 - H(y,r)(\dot{y}^2 + \dot{r}^2)} = -\tau_p V \int dt \mathcal{L} \\ \mathcal{L} &= \sqrt{\frac{1}{H} - \dot{y}^2 - \dot{r}^2 - r^2 \dot{\theta}^2} \end{aligned} \tag{64}$$

Therefore, one may take $H^{-1}$ as the effective potential for the scalar fields. It is not hard to find out that $r = 0, y = 2n\pi r_0$ is at the minima of the potential and is stable, while $r = 0, y = (2n+1)\pi r_0$ is a saddle point, where $r = 0$ is stable but $y = (2n+1)\pi r_0$ is unstable with tachyonic fluctuations. In [6], it was argued that when the Dp-brane stay at the saddle point, it behaves as an unstable non-BPS brane and there exist a kink solution which makes (Dp, NS5)'-system BPS after tachyon condensation. More precisely, one of the Dp-brane worldvolume direction changes to the compactifed $y$ direction so that the overlap dimensions between Dp-brane and NS5-branes are $p$ rather than $p+1$. This is a very interesting observation and shed new light on the study of non-BPS brane. But actually, the tachyon condensation could have many other channels besides forming kink solution. It is very possible that the Dp-brane still falls to the NS5-branes from $y = \pi r_0$ to $y = 0$. Also even we fix at $y$ direction, the radial behavior of Dp-brane in the three noncompactified directions is quite interesting in its own right. In this section, we would like to address this issue from the analysis of DBI action.

Note that for simplicity, we turn off all the electromagnetic fields. When we discuss the homogeneous evolution, the electric field is always a constant. It is straightforward to take into account the contribution of the electric field, as we have done in the last section.

Before we turn to the analysis of the tachyon condensation without forming kink, we would like to point out that there is an issue on the gauge dynamics of the kink solution. In [16, 17], it has been shown how to

construct the fundamental strings ending on the BPS kink solutions in the nonBPS brane effective action. After field redefinition, it is easy to analyze the same issue in the gauge dynamics of kink solution of (Dp, NS5)'-system.

The density of canonical momentum and Hamiltonian are

$$\begin{aligned} \mathbf{\Pi}_\theta &= \frac{\tau_p r^2 \dot{\theta}}{\mathcal{L}}, \qquad \mathbf{H} = \frac{\tau_p}{\mathcal{L}H} \\ \mathbf{\Pi}_y &= \frac{\tau_p \dot{y}}{\mathcal{L}}, \qquad \mathbf{\Pi}_r = \frac{\tau_p \dot{r}}{\mathcal{L}} \end{aligned} \tag{65}$$

and the equations of motions read as:

$$\begin{aligned} \mathbf{\Pi}_\theta &= L \\ \mathbf{H} &= E \\ \dot{\mathbf{\Pi}}_\mathbf{y} &= -\frac{\tau_p k l_s^2 \sinh\frac{r}{r_0}\sin\frac{y}{r_0}}{4\mathcal{L}H^2 r_0^2 r(\cosh\frac{r}{r_0}-\cos\frac{y}{r_0})} \\ \dot{\mathbf{\Pi}}_\mathbf{r} &= \frac{\tau_p}{2\mathcal{L}}\left(\frac{k l_s^2}{2H^2 r_0 r(\cosh\frac{r}{r_0}-\cos\frac{y}{r_0})^2}\left(-\frac{\sinh\frac{r}{r_0}(\cosh\frac{r}{r_0}-\cos\frac{y}{r_0})}{r}+\frac{1-\cos\frac{y}{r_0}\cosh\frac{r}{r_0}}{r_0}\right)+\frac{2L^2}{r^3E^2H^2}\right) \end{aligned} \tag{66}$$

where $L$ and $E$ are conserved angular momentum and energy respectively.

It is also instructive to analyze the strength tensor. From [9], we have

$$T^{\mu\nu} = -\tau_p \frac{\sqrt{-detG}}{\sqrt{H}} \begin{pmatrix} \frac{1}{det(G)} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = -\tau_p \mathcal{L} \begin{pmatrix} \frac{-1}{H\mathcal{L}^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{67}$$

So the pressure is

$$p = -\tau_p \mathcal{L} = -\frac{\tau_p}{EH} \tag{68}$$

Generically, the equations of motions (66) are too involved to be solved exactly. Nevertheless, there are several cases that one can deal with to get a qualitative picture of the dynamics. It's clear from (66) that when we set $y = n\pi r_0, \dot{y} = 0$ as initial condition, the Dbrane will not evolve in the $y$ direction, no matter how it evolves in the other transverse directions. Firstly we fix $y = \pi r_0$ and the equation of motion of $r$ can be cast into an effective one-dimensional problem:

$$\dot{r}^2 = -V_{eff} = \frac{1}{H} - \frac{1}{E^2H^2}(\tau_p^2 + \frac{L^2}{r^2}) \tag{69}$$

and the equation on $\theta$ is

$$\dot{\theta} = \frac{L}{r^2 EH} \tag{70}$$

In practice, one can learn the qualitative feature of the radial dynamics from the effective potential. It's illuminating to see its behavior at large and small $r$, compared to $r_0$. Notice that the minimal length of $r_0$ cannot be smaller than the string scale $l_s$.

When $r/r_0$ is large, up to order $\frac{l_s}{r}$, $H = 1 + \frac{kl_s^2}{2r_0 r}$, and

$$V_{eff} = (\frac{\tau_p^2}{E^2} - 1) + \frac{kl_s^2}{2r_0 r}(1 - \frac{2\tau_p^2}{E^2}) + \frac{L^2}{E^2 r^2} + O(\frac{1}{r^3}) \tag{71}$$

where we omit the terms proportional to $(\frac{kl_s^2}{2r_0 r})^2$.

When $r/r_0$ is small, $H = 1 + \frac{kl_s^2}{4r_0^2} - \frac{kl_s^2 r^2}{12 r_0^4} + O(k(\frac{l_s}{r_0})^2(\frac{r}{r_0})^{2+\varepsilon})$ $(\varepsilon > 0)$. After neglecting square and higher order of $(\frac{r}{r_0})^2$, one can take $H$ as a constant $H_0 = 1 + \frac{kl_s^2}{4r_0^2}$ and

$$V_{eff}(r) = (\frac{\tau_p^2}{E^2} - 1) + \frac{L^2}{E^2 r^2} \tag{72}$$

Therefore we see that in both cases, the effective potential is well approximated by

$$V_{eff}(u) = au^2 + bu + c \tag{73}$$

with $u = \frac{1}{r}$ and

$$\begin{aligned} a &= \frac{L^2}{E^2}, \;\; b = \frac{kl_s^2}{2r_0}(1 - 2\frac{\tau_p^2}{E^2}), \;\; c = \frac{\tau_p^2}{E^2} - 1, \quad \text{for large } r/r_0 \\ a &= \frac{L^2}{E^2 H_0^2}, \;\; b = 0, \;\; c = \frac{\tau_p^2}{E^2 H_0^2} - \frac{1}{H_0}, \quad \text{for small } r/r_0 \end{aligned} \tag{74}$$

Depending on the parameters, the equation $V_{eff}(u) = 0$ could have two different roots $u_1 < u_2$, one root $u_0$ or no root. In fact, the one-dimensional problem we have is quite similar to the radial motion of an object in a Newtonian gravitational system. Remarkably, this effective potential captures the qualitative feature of the exact effective potential in (69). In Fig. 1, through numerical analysis, we draw off the relation between $V_{eff}$ and $r$, by adjusting the conserved quantity $E$ and $L$. From the form of the potential, one can read out the possible radial motions of the D-brane. We will see that the four possibilities could be compared with the semi-quantitative analysis in the large $r/r_0$ region.

Let us discuss the radial behavior case by case. When $r/r_0$ is large, we have

(1) when the angular momentum $L = 0$, if $E < \tau_p$, the D-brane moves in a restricted region $0 < r < r_m$ with $r_m = 1/u_0$; otherwise it could escape to infinity;

(2) when $L \neq 0$, and $b^2 - 4ac > 0$, $c > 0$, there is an elliptic orbit, and the pressure $p \sim \frac{1}{H}$ is a monotonically increasing function of $r$, but varies slowly. In terms of $E, L$, we require $E < \tau_p$ and

$$\frac{L^2}{E^2} < \frac{(1+2c)^2}{4c}(\frac{kl_s^2}{2r_0})^2, \tag{75}$$

such that the D-brane move between $r_{min}$ and $r_{max}$, where $r_{min,max}$ are the inverse of roots of $V_{eff} = 0$.

(3) when $L \neq 0$, and $b^2 - 4ac > 0$, $c < 0$, namely $E > \tau_p$, Dbrane is moving in the range $(r_{min}, \infty)$, where $r_{min} = 1/u_2$ is the inverse of the positive root of (73). The pressure $p$ increase as $r$ increase, getting to asymptotically $\frac{\tau_p}{E}$ when $r$ large.

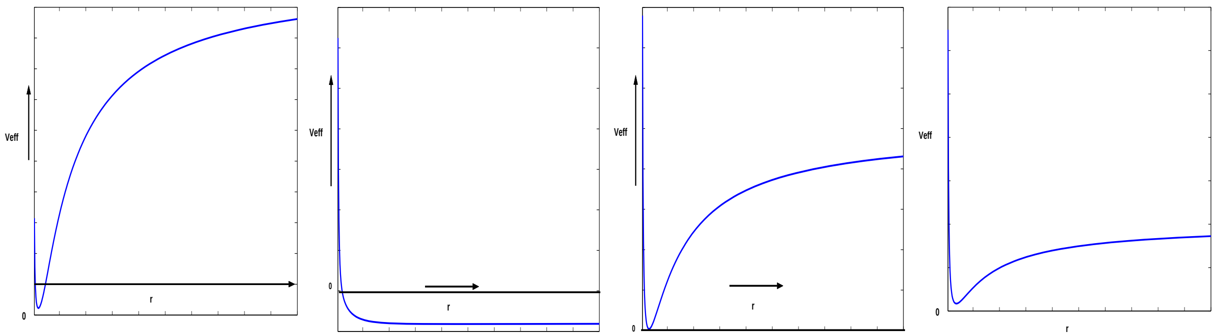


Figure 1: y fixed at $\pi r_0$, case(2)-(5). These figures are obtained by numerical analysis from the original potential (69), and is semi-quantitatively same as (73).

(4) when $L \neq 0$, and $b^2 - 4ac = 0$, namely $E < \tau_p$ and

$$\frac{L^2}{E^2} = \frac{(1+2c)^2}{4c}(\frac{kl_s^2}{2r_0})^2, \tag{76}$$

there is an circle orbit, the pressure is constant. The radius of the circle is

$$r_c = \frac{1+2c}{2c}\frac{kl_s^2}{2r_0}, \tag{77}$$

so that in order to have circular orbit in the large $r/r_0$ region, one needs to require $k$ be very large or $E \simeq \tau_p$;

(5) $b^2 - 4ac < 0$ is not physical, actually it is inconsistent with (65).

In short, when $E < \tau_p$, the D-brane could not escape to infinity and move in a restricted region. In particular, the angular momentum play its role in a subtle way: when $L$ satisfies (76) there exist a stable circular orbit; when $L$ satisfies (75), there exist an elliptic orbit; otherwise, there is no motion at all. Qualitatively, the case (2)-(5) could be mapped to four cases in Fig 1..

On the other hand, we are more interested in the radial behavior of the D-brane when $r$ near its stable point, i.e. $r/r_0 \sim 0$. In this case, the pressure is a constant and

(1) when $L = 0$, $E > \frac{\tau_p}{\sqrt{H_0}}$, it is of linear motion with constant velocity, and could escape to infinity, while $E \leq \frac{\tau_p}{\sqrt{H_0}}$ is inconsistent with (65), actually it is obvious from (65) that $E \geq \frac{\tau_p}{\sqrt{H_0}}$.

(2) when $L \neq 0$, $E > \frac{\tau_p}{\sqrt{H_0}}$, it could move between $1/u_2$ and $\infty$, $E \leq \frac{\tau_p}{\sqrt{H_0}}$ is unphysical as stated above.

Now the evolution of the Dp-brane is very different from the uncompactified case. First notice that one has a limit on the energy $E \geq \frac{\tau_p}{\sqrt{H_0}}$. More interestingly, we have $r > \frac{L}{E\sqrt{H_0}}$, which means that due to the existence of the angular momentum, the D-brane cannot reach $r = 0$.

In both regions, we have $r^2\dot{\theta} \simeq L$. So the evolution is much the same as that of a particle in Newtonian gravitational field.

Let's try to combine the discussions in the two regions together to have a picture of the Dp-brane radial motion: when $L = 0$ and $\tau_p > E > \frac{\tau_p}{\sqrt{H_0}}$, the Dp-brane could move in a restricted region and get to $r = 0$; when $L \neq 0$ and $\tau_p > E > \frac{\tau_p}{\sqrt{H_0}}$, the D-brane may still move in a restricted region, but could never reach $r = 0$; when $E > \tau_p$, the D-brane can escape to infinity.

If we instead fix $y = 2n\pi r_0$, the large $\frac{r}{r_0}$ behavior is the same as above, but for small $\frac{r}{r_0}$,

$$\begin{aligned} H &= 1 + \frac{kl_s^2}{2r_0 r}\frac{\sinh\frac{r}{r_0}}{\cosh\frac{r}{r_0} - 1} \\ &= 1 + \frac{kl_s^2}{r^2} + O(k(\frac{l_s}{r_0})^2(\frac{r}{r_0})^{2+\varepsilon}) \quad (\varepsilon > 0) \\ &\simeq \frac{kl_s^2}{r^2} \end{aligned} \tag{78}$$

where in the last step, we use the near throat region approximation. Obviously, it looks the same as the one in uncompactified case[5]. Again we obtain approximate effective potential that is semi-quantitatively consistent with numerical analysis. Thus

$$V_{eff} = \frac{\tau_p^2}{k^2 l_s^4 E^2} r^4 + \frac{L^2}{k^2 l_s^4 E^2} r^2 - \frac{1}{kl_s^2} r^2 \tag{79}$$

In this case we have if

(1) $\frac{L}{E} < \sqrt{k} l_s$ and $E < \tau_p$, the Dbrane is bounded near NS5 branes in the range $(0, r_{max})$, and when it get close to NS5 branes, the pressure decrease as $r^2$;

(2) $\frac{L}{E} > \sqrt{k} l_s$ and $E < \tau_p$, this is forbidden in the near throat region;

(3) $\frac{L}{E} < \sqrt{k} l_s$ and $E > \tau_p$, the Dbrane is moving in the range $(0, \infty)$, the pressure behaves as $\frac{\tau_p r^2}{E k l_s^2}$ and increases very slowly as $r$ becoming larger, getting to asymptotic value $p_\infty = \frac{\tau_p}{E}$;

(4) $\frac{L}{E} > \sqrt{k} l_s$ and $E > \tau_p$, there is a potential barrier near NS5 brane, so the Dbrane will be scattered and can never reach the near throat region. The pressure increases very slowly as $r$ becoming larger, getting to asymptotic value $p_\infty = \frac{\tau_p}{E}$.

In the near throat region, one has to require $L < \sqrt{k} l_s E$. In other words, if $L \geq \sqrt{k} l_s E$, the D-brane cannot enter the near throat region. Actually, one can solve the equation exactly and get the small $r/r_0$ behavior like (43) but now

$$\alpha^2 = \frac{1}{kl_s^2}(1 - \frac{L^2}{kl_s^2 E^2}) \ , \quad \beta^2 = \frac{kl_s^2 E^2}{\tau_p^2}(1 - \frac{L^2}{kl_s^2 E^2}). \tag{80}$$

We see that $r$ and pressure decrease exponentially. For $y$ fixed at $2n\pi r_0$, when $r$ is large, we still have $\dot{\theta} \simeq \frac{L}{r^2 E}$, but when $r$ is small $\dot{\theta} \simeq \frac{L}{kl_s^2 E}$, that is $\theta \propto t$, keeping the angular momentum conserved.

Now let's consider more generally initial conditions. From (66), we see that when $0 < y < \pi r_0$, $\dot{\mathbf{\Pi}}_{\mathbf{y}} < 0$ and when $\pi r_0 < y < 2\pi r_0$, $\dot{\mathbf{\Pi}}_{\mathbf{y}} > 0$. This is nothing but the fact that $y = \pi r_0$ is at the peek of the potential and $y$ tends to roll down to $y = 0$.

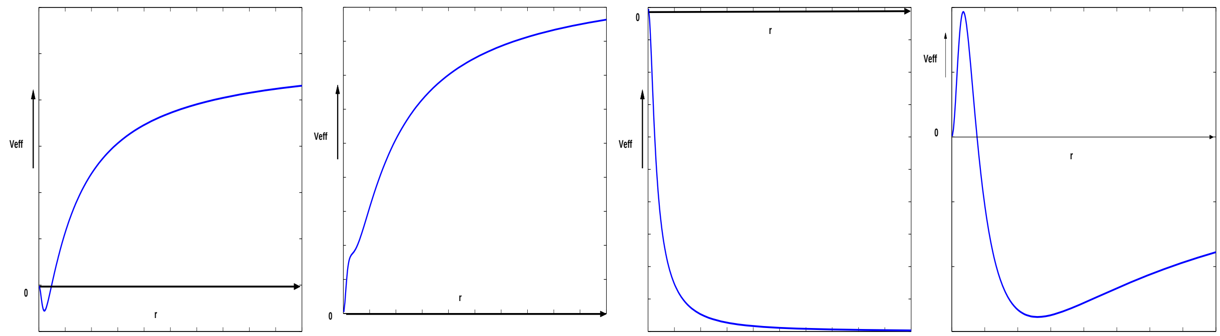


Figure 2: y fixed at $2n\pi r_0$, case(1)-(4). These figures are obtained by numerical analysis from the original potential (69), and is semi-quantitatively same as (79).

For the $r$ direction, the behavior is even more complicated. When $y = \pi r_0$ we have

$$\dot{\mathbf{\Pi}}_r = \frac{\tau_p}{2\mathcal{L}H^2}\left(\frac{kl_s^2}{2rr_0(\cosh\frac{r}{r_0}+1)}\left(-\frac{\sinh\frac{r}{r_0}}{r}+\frac{1}{r_0}\right)+\frac{2L^2}{r^3E^2}\right) \tag{81}$$

It is clear that in this case, when $L = 0$, $\dot{\mathbf{\Pi}}_r < 0$, or say r=0 is the minimum of potential. But nonvanishing angular momentum causes centrifugal force which dominates when r is small($\sim \frac{1}{r^3}$). And when y is near the unstable saddle point $\pi r_0$, D-brane feels repulsive force in near throat region, and attractive force in far away region. This fact is consistent with our argument that when $y = \pi r_0$, the nonvanishing angular momentum forbids the Dp-brane from approaching $r = 0$.

When $y = 2n\pi r_0$ we have

$$\dot{\mathbf{\Pi}}_r = \frac{\tau_p}{2\mathcal{L}H^2}\left(-\frac{kl_s^2}{2rr_0(\cosh\frac{r}{r_0}-1)}\left(\frac{\sinh\frac{r}{r_0}}{r}+\frac{1}{r_0}\right)+\frac{2L^2}{r^3E^2}\right) \tag{82}$$

Again there is an attractive force and a repulsive centrifugal force from angular momentum. The former dominate in the large $r/r_0$ region, but in the near throat region the competition depends on $\alpha = \frac{L^2}{kl_s^2E^2}$. Recall that one requires $L < \sqrt{k}l_sE$ so that $\alpha < 1$ and the total force is still attractive.

Therefore, if we assume that the Dp-brane initially deviate from $y = \pi r_0$ a little bit, it will roll down to the stable point $y = 0$. Let's see what is the radial motion of D-brane in the three uncompactified directions. We have

$$\dot{y}^2 + \dot{r}^2 = -V_{eff} = \frac{1}{H} - \frac{1}{E^2H^2}\left(\tau_p^2 + \frac{L^2}{r^2}\right) \tag{83}$$

In the large $r/r_0$ region, $H = 1 + \frac{kl_s^2}{2r_0r}$ is independent of $y$. So as $y$ rolls down, the orbit of the radial motion changes if $\dot{y}$ is not a constant. But if $E < \tau_p$, the D-brane move in a restricted region and cannot escape to infinity. It could be possible that the D-brane move in an elliptic or circular orbit and never come near to the near throat region when $E \simeq \tau_p$. If the D-brane can move into the near throat region with $L < \sqrt{k}l_sE$, it may roll to $r = 0$ since at $y = 0$ the D-brane feel attractive force. However, the dynamics here is really complicated and depends on the initial conditions. For example, even when $y$ rolls to $y = 0$ but with nonzero $\dot{y}$, the D-brane will move in a restricted region rather than reaching $r = 0$ directly. As $y$ oscillate around $y = 0$ and finally frozen to $y = 0$, the D-brane will condensate to $y = 0$ at the end.

The gauge dynamics around the tachyon vacuum is very similar to the one without compact transverse dimension. At the end of the tachyon potential, the Hamiltonian could be rewritten as

$$\mathcal{H} = \sqrt{\mathbf{\Pi}_e^2 + \frac{\mathbf{\Pi}_y^2}{H} + \frac{\mathbf{\Pi}_r^2}{H} + \frac{\mathbf{\Pi}_\theta^2}{r^2 H} + \frac{\tau_p^2}{H}}. \tag{84}$$

Obviously, the energy consists of the kinetic energy of $y$ and $r$, the flux energy and the one from angular momentum. The gauge dynamics could be studied in the same way as in last section. The key point is to treat the field $y$ also as a tachyon field component and the dynamics could still be cast into a fluid equation. We will not repeat it here.

# 4 Conclusions and Discussions

In this paper, we investigated in details the radial motion of Dp-brane near NS5-branes without and with one compactified transverse direction, using the DBI effective action of (Dp, NS5)-system. In the case without compactified dimension, we studied the dynamics with the world volume gauge fluxes and the angular momentum. We mainly focused on the homogeneous evolution, in which case the electric fields on the D-brane are conserved. With various initial conditions, the D-brane could move in a restricted region or escape to infinity. But there is no stationary orbit. The influence of the gauge field on the dynamics could be studied in the Hamiltonian formulation. At the end of the tachyon condensation, the pressureless remnants has two components, a string fluid and the tachyon matter. Around the tachyon vacuum, the Hamiltonian dynamics of our geometric tachyon system is very similar to the one in the rolling tachyon case, after simple field redefinition. The dynamics could be described by an effective fluid equations augmented by integrability condition. Various issues in the rolling tachyon case could be carried over to our geometric tachyon configuration. Even taking into account of the angular momentum, the product of the tachyon condensation is still pressureless. The angular momentum will slow down the falling the Dp-brane and lowers the critical value of the electric field.

In the case with one transverse dimension compactified, we also study D-brane dynamics with various initial conditions, both in the large and small radial distance regions. The dynamics is different if we initially put the D-brane at the meta-stable point($y = \pi r_0$, $r_0$ the compactification radius) or at the stable point($y = 2n\pi r_0$). The former case shows quite different features from the uncompactified case. By appropriate approximation, the radial motion is equivalent to a point particle moving in a Newtonian potential. It is familiar that in this case one can have elliptic orbit, circular orbit, and unbounded orbit corresponding to different initial conditions. Due to the existence of the angular momentum, the Dp-brane cannot reach $r = 0$. In the latter case, the dynamics is very reminiscent of that of uncompactified case, the radial coordinate as well as the pressure falls exponentially with time in the near throat region. If initially $y$ deviate from $y = \pi r_0$, it will rolls to $y = 0$ and the radial motion of D-brane in the three uncompactified directions depends on the initial conditions. If $E < \tau_p, L < \sqrt{k} l_s E$, the radial motion could be stable orbit in the large $r$ region, but more possibly it rolls to $r = 0$ at the end.

It might be very interesting to investigate what these dynamics corresponds to in the Little String

Theory. In [8], it was argued that the D-brane could be taken as the defect in the dual LST. It is not clear how to describe the dynamics here in the dual picture. In [4, 15], in the framework of the DBI effective action, the open/close duality has been investigated in some details. It would be nice to understand this issue in (NS5, D)-system. It also deserves further study if we can have a stringy treatment beyond effective action of the various dynamical processes.

As we have shown in detail, the radial dynamics of the (Dp NS5)-systems are very similar to the rolling tachyon, but it also has something new. The input of the angular momentum make the dynamics of the (Dp NS5)-systems much richer. It would be interesting to study the inhomogeneous evolution of the system as have been done in the tachyon case[19].

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